# Maths Calculation Strategies

Foundation Stage, Key Stage 1 and 2



A guide for parents

# INTRODUCTION

This booklet is designed to help you understand the calculation strategies your child will encounter in maths, to help you support them with their learning and has been updated in line with the New National Curriculum 2014.

In this booklet you will see a variety of ways of working out different calculations.

It explains the strategies used for mental and written calculations in schools within our Catholic Pyramid from The Early Years Foundation Stage to Year 6. By the end of year 6, children will have been taught a range of calculation strategies, both mental and written. They progress through each step when they are ready and confident to do so.

All calculations should be written horizontally at first, until children are taught vertical methods e.g.

45 + 13 =	<u>NOT</u> vertically	45
		+13

It is important that children are taught and use correct mathematical vocabulary. They become familiar with the words, calculation and calculate as they are used frequently in school. The word 'sum' should only be used when adding numbers together.

It is important to use the correct terminology (words) when talking about the numbers in calculations. The numbers are said using the value of the digit, for example;

<b>4</b> 5	
+ <mark>1</mark> 3	
<b>5</b> 0	add the tens by saying <b>forty</b> add <b>ten</b> is <b>fifty</b> , <b>not 4 and 1</b> = <b>5</b>

Children should be encouraged to estimate their answers before calculating them. It is also important that they are encouraged to consider whether they can use a mental strategy to solve a calculation before using a written method to be more efficient.

The calculation methods used by children today are in many cases different from those used by adults when they were at school. This can cause anxiety, with parents and carers unsure whether or not they should teach children particular methods of calculation.

Therefore the aim of this booklet is to support you as parents and carers to understand the methods of calculation that your children are being taught here at school and which they are familiar with in order for support to be given at home, which reinforces what is being taught in the classroom.

This booklet is divided into each of the 4 operations (addition, subtraction, multiplication and division) and shows the progression of the methods taught from Year 1 through to Year6.

For further clarification of how to use any of these methods of calculation or which specific methods of calculation your child is currently being taught, please contact your child's class teacher.

I hope this booklet proves to be of help in providing your child with continuity in their Maths learning both at home and school.

Mrs Copeman

# ADDITION

## Addition - Mental Methods

Children are encouraged to develop a mental picture of numbers. They develop ways of recording calculations using pictures and drawings.



Children are taught to understand addition as combining two or more sets and counting on.

2 + 3 = 🗆

At a party, I eat 2 cakes and my friend eats 3. How many cakes did we eat altogether?



Children could draw a picture to help them work out the answer.

8 + 4 = 🗆

8 people are on the bus. 4 more get on at the next stop. How many people are on the bus now?



Children could use dots or tally marks to represent objects (this may be quicker than drawing pictures)

Practical resources and familiar objects are used to support addition skills. The children learn to count and add in ones at this stage. They sing songs, listen to stories and play games.

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Numicon

8 + 5 = 13



## Addition - Informal Written Methods

This is an empty number line. It is useful and important for developing children's mathematics.

The line has no markings or scale. The empty number line is designed to allow children to create an image or picture of a calculation in their minds. This can then help improve their mental calculations.



We use the empty number line for addition in the following ways.

#### The 'tens jumping' method (Introduced in Year 2)

(Jumping along the line in multiples of ten from the starting number)



We keep the 34 whole and write it at the beginning of the number line. Then jump in multiples of 10 to 44, then 54 to add the 20. Then make a jump of 3 (so that in this case 23 has been added altogether) reaching 57.

This method is developed further by adding the tens in one jump and the units in one jump when the children are able to do so. (Introduced in Year 2)



As it is easier to add multiples of 10, we round the smaller number in the calculation up to the next multiple of ten (we round 49 up to 50 in this case) and make an 'over jump'.

Here we have begun by writing 73 at the beginning of the number line (as it's the larger of the two numbers in the calculation). We add the 50, which lands us on 123. We then jump back the amount of units too many that we added, in this case back 1 as we added 50 and were supposed to be adding 49. This gives us the answer 122.

### Addition - Formal Written Methods

The next progression following the use of an empty number line for addition is for children to learn about the value of each of the digits and to partition numbers (break them into Hundreds, Tens and Units) and begin to use vertical methods of calculation.

<u>Addition using the partitioning method (</u>Introduced in Year 2)

142 + 119 = 261  $H \quad T \quad U$  100 + 40 + 2  $+ \quad 100 + 10 + 9$  200 + 50 + 11 = 261

Both numbers are partitioned into H T and U. They are placed above one another in columns. The numbers in each column are then added together and the totals are written below the corresponding column. These totals are then added together to get the total.

In this example 100 + 100 = 200, 40 + 10 = 50 and 2 + 9 = 11. Then 200 + 50 + 11 = 261

Expanded method of addition (Introduced in Year 3)

Firstly the numbers are written one above the other with the HTU in the correct columns. Then the Units are added and the total is recorded below the line (2 + 1 = 3). The tens are then combined and the total is written below the units total (40 + 60 = 100). Next the hundreds digits are combined and the total is written below the tens total (100 + 200 = 300). Finally these 3 totals are combined to get the final answer to the calculation (3 + 100 + 300 = 403). 789 + 642 = **143**1

	Н	Т	U
	7	8	9
+	6	4	2
1	4	3	1
	1	1	

Firstly the numbers in the calculation are written one above the other with the HTU digits in the correct columns. The Units are added first and the total is recorded below the answer line. If the total of the units is greater than 9, the units digit is written down and the tens digit is carried and placed below the tens column. This will then be added when the tens column is totalled. The number of tens is totalled and noted. If the total of tens is greater than 9 then each lot of 10 tens (making 100) is carried into the hundreds column. This would continue if calculating with 4 or more digits.

In this example 9 + 2 = 11. We write the 1 in the units column and carry the 1 (ten) into the tens column. Next you add 8 (tens) + 4 (tens) = 12 (tens) and add on the 1 (ten) you carried into that column making 13 (tens) altogether. You note the 3 (tens) in the answer line and carry the 1 (hundred...which is 10 tens) into the hundreds column. You then add 7 (hundreds), 6 (Hundreds) and the 1 (hundred) that you carried over, totalling 14 (hundreds). This is then written in the answer line making the final total **1431**.

This same method is used when adding decimal numbers (Introduced in Year 4)

2.71
+ 42.42
45.13
1

# SUBTRACTION

## Subtraction - Mental Methods

Children are taught to understand subtraction as taking away (counting back) and finding the difference (counting up). They are introduced to subtraction by counting physical objects and taking a specific number away. Children sing number songs, listen to stories and play number games.





If I had 3 teddies and I take away 2 teddies, how many did I have left?

3 - 2 = 🗆

A teddy bear costs £5 and a doll costs £2. How much more does the bear cost?



Find the difference.

Drawing a picture helps children to visualise the problem.

8 - 3 = D Mum baked 8 biscuits. I ate 3. How many were left?

Lisa has 8 felt tip pens and Harry has 3. How many more does Lisa have?

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• • • Find the difference
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Bead strings or bead bars can be used to illustrate subtraction including bridging through ten (moving into the next lot of 10).

In this example below begin by first counting back 3 (red beads) then counting back 2 (white beads) to have counted back 5 altogether.



# Subtraction - Informal Written Methods

Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

12 - 3 = 9



This progresses to then using the empty number line in the following ways for subtraction calculations.

The 'tens jumping' method (Introduced in Year 2)



This method takes away multiples of ten at a time by jumping backwards, first in multiples of ten to 27 (2 jumps of ten in this example), then back 3 to 24 (to have taken away 23 in total).

Subtraction by 'Counting on' (Introduced in Year 2)

This method **<u>can only</u>** be used for subtraction.

In order to use this method children must understand that subtraction does not only mean 'taking away' but that it can also mean 'difference'.

When using the number line, addition is always carried out moving from left to right as the total gets larger. And subtraction is carried out from right to left as the total gets smaller.

However, because the 'Counting on' method is requiring the children to find the difference between the two numbers, it is carried out from left to right on the number line.

The 'counting on' method using a number line. (Introduced in Year 2)



Put both 43 and 28 on the number line. You are then 'counting on' from the smaller to the larger number. Starting at 28, you count on to the next multiple of 10, in this case from 28 to 30. From the multiple of ten you then count on in multiples of 10 to the multiple of 10 in the number you are trying to reach, in this case 40 (the multiple of ten in the number  $\underline{4}$ 3). The final step is to add what is remaining to get to the target number, in this case adding 3 to get from 40 to 43. The total of all the small jumps made is your answer, in this case 2 + 10 + 3 = 15.

# Subtraction - Formal Written Methods

Expanded method of decomposition (without exchanging) (Introduced in Year 3)

		ΗΤυ
289 -157	=	200 + <mark>80</mark> + 9
107		<u>-100 + 50 + 7</u>
		100 + <mark>30</mark> + 2 = 1 <mark>3</mark> 2

The children learn to partition the number into its place value parts, such as hundreds, tens and units. They then start subtracting the least significant digits first (the units) The units in the second number in the calculation are subtracted from the units in the first number (9-7 = 2 in this example), the tens from the tens (80 - 50 = 30 in this example) and the hundreds from the hundreds (200-100 = 100 in this example). For each one of these stages in the process, the answer is written under the answer line in the corresponding column. Once this has been completed the answers are combined to find the total (100 + 30 + 2 = 132 in this example)



In the example of the expanded method above, there is the need to 'exchange' (In the past this was called 'borrowing' but the term 'exchange;' is now used)

In this example you begin looking at the units and as 4 is less than 7 you need to exchange the 60 + 4 into 50 + 14. You can then subtract 7 from 14. You then subtract 30 from 50 and 200 from 300.

The children will also use this method with 2 digit numbers. The children need to be aware that they **DO NOT** just subtract the smaller number from the greater number it must always be the bottom layer of the calculation (when set out vertically) taken away from the number above.

From the expanded method of decomposition, and having developed an understanding of the need to 'exchange' the children then learn the standard decomposition method below.

#### Decomposition (Introduced in Year 3)

<u>With 3 digit numbers</u>	With 4 digit numbers	With decimal numbers
<b>93</b> 2 - 4 <b>5</b> 7 = 4 <b>7</b> 5	<mark>2</mark> 314 - <mark>1</mark> 425 = 889	231.44 - 161.25 = 70.19
ΗΤυ	<mark>Тh</mark> H T U	H T U. t <mark>h hth</mark>
$^{8}$ $^{12}$ $^{1}$ $^{2}$ $^{2}$ $^{2}$	1 12 10 1 <del>2 3 1</del> 4	$ \begin{array}{ccccccccccccccccccccccccccccccccc$
- <u>4 5 /</u> 4 7 5	<u>-1425</u>	<u>-161.25</u>
	889	70.19

Now the children are secure with understanding why and how to exchange numbers they are able to use this standard method of decomposition where the numbers are represented by a digit, e.g. they understand that the digit 5 in 457 represents 50.

When using this method of subtraction again they begin subtracting the least significant digits (Units), followed by tens, hundreds, thousands etc

For example, in the 3 digit number example above the calculation has been solved by realising that 7 (units) can not be taken away from 2 (units) so 1 ten from the 3 (tens) has been exchanged to leave 2 (tens) and allow 2 (units) to become 12 (units). This enables us to do 12-7 = 5. The 5 is recorded in the answer line. You are now left with 2 (tens) which you write as a small 2 when you cross out the 3 (tens). You can't subtract 5 (tens) from 2 (tens) so we exchange 1 (hundred...which is also 10 tens) and enables us to have 12 (tens) in the tens column. We cross out the 9 and write a small 8 to represent the 8 (hundreds) we are left with in the column and write a small 1 in the tens column to represent this exchange. We can now do 12 (tens) subtract 5 (tens) giving us 7 (tens). This 7 is recorded in the answer line in the tens column. Finally we then subtract 4 (hundreds) from the 8 (hundreds) that we have left, giving us 4 (hundreds) and this is recorded in the answer line in the hundreds column. This means the answer to our calculation is 475

The method is carried out in the exact same way whether you are using 2, 3, 4 or more digit numbers and also those which have decimals in them too.

# MULTIPLICATION

# Multiplication - Mental Methods

Early multiplication skills begin in Early Years with counting in different sized steps. Children are required to find doubles and halves of numbers.



Children count equal groups of objects and repeated groups of the same size. We refer to this as repeated addition. As with addition and subtraction a picture can be helpful so that the children can visualise the calculation to begin with.



They count in 2s, 5s and 10s and work on practical problem solving activities involving equal sets or groups. They can say whether sets are the same, larger or smaller.

A strategy to help children learn multiplication tables facts from counting is to say or show the child a multiplication fact such as:

5 x 2 =

Ask the child to put up five fingers and count across the five fingers in multiples of two. Five lots of 2 is 10.

The children do need to understand that  $1 \times 2 = 2$ ,  $2 \times 2 = 4$ ,  $3 \times 2 = 6$  etc so that they understand what the numbers relate to, not just recall a string of numbers, e.g. 2, 4, 6, 8, with no understanding what the number relates to.

# Multiplication - Informal Written Methods



<u>Arrays</u> (Introduced in Year 1)



It is important that children understand that multiplication is commutative meaning that  $4 \times 5$  will give the same answer as  $5 \times 4$ .

We can explain this visually by using an array so that the children can see totalling up three rows of 4 gives f the same total as four columns of 3.

Using arrays also helps children to understand the relationship between multiplication and division. In this example:

 $12 \div 3 = 4$  and  $12 \div 4 = 3$   $3 \times 4 = 12$  and  $4 \times 3 = 12$ 

There are 5 cakes in a pack. How many cakes are there in 3 packs?



#### Using the number line for multiplication

<u>Repeated addition (Introduced in Year 2)</u>



There are 4 cats. Each cat has 6 kittens. How many kittens are there altogether?



Repeated addition using a number line requires children to count on in equal steps, recording each jump on the number line. In this case making 4 steps, adding 6 each time.

Partitioning (Introduced in Year 3)

TU  $14 \times 6 = (10 \times 6) + (4 \times 6)$  = 60 + 24= 84



To use this partitioning method for multiplication, any number in the calculation with more than one digit is partitioned into Tens and Units etc

In this example 14 is partitioned into 10 and 4. These numbers are then separately multiplied by the other number in the calculation (6 is this case)

In this example

 $10 \times 6 = 60$  so the number 60 is written below the calculation.

 $4 \times 6 = 24$  so the number 24 is written next to the answer to the previous calculation.

These are then added together 60 + 24 to give us the answer **84**.

# Multiplication - Formal Written Methods

#### The Grid method (Introduced in Year 3)

The grid method uses the idea of partitioning larger numbers and combines this with setting the numbers out in a grid to solve a multiplication calculation.



In this example the 25 has been partitioned into tens and units, **20** and **5** and the grid is used to record the calculation. The 20 and 5 are put along the top of the grid and the 6 that we are multiplying by down the side.

The calculation is then carried out in 3 steps.

20 x6 = 120

**5** x 6 = 30

Both of these answers are recorded in the corresponding rectangle in the grid.

Finally the 120 and 30 and combined to get the total = 150

This method can be used to solve calculations using three and four digits numbers and decimals too e.g. TU  $\times$  TU or HTU  $\times$  TU etc

Further examples of using this method with larger numbers

#### 34 x 51 = 1734

х	30	4	
50	<b>150</b> 0	200	= 1700
1	30	4	= 34

<u>HTU x U</u>

## 346 x 9 = <mark>311</mark>4



2700 + 360 + 54 = <u>3114</u>

<u>HTU x TU</u>

152 x 29 = 4408



= <u>4408</u>

#### Short multiplication (Introduced in Year 4)



First you multiply the least significant digits 'units' ( $4 \times 6 = 24$ ) The 4 is recorded in the answer line and the two (tens) in the number 24 are carried forwarded and placed under the 'T' column ready to be added into the next calculation. Then the tens are multiplied by the unit ( $20 \times 6 = 120$ ) You need to add on the two (tens) that you carried forward from the previous calculation which totals 140. The digits are written in the correct columns to give the answer 144.

The same applies when using this method to multiply larger numbers.

342 x 7 = 2394 2741 x 6 = 1644							16					
Th	Н	т	U				Т	Th	Th	Н	Т	U
	3	4	2						2	7	4	1
<u>X</u>			7				<u>X</u>					6
2	3	9	4					1	6	4	4	6
	2	1							4	2		

## Long multiplication (Introduced in Year 5)

24	x 1	6 =	384
	Н	Т	U
		2 <b>2</b>	4
X		1	6
	2	4	0
	1	4	4
	3	8	4

When using the long multiplication method you begin by multiplying the first number in the calculation (in this case 24) by the most significant digit in the second number (in this case the tens digit in the number  $\underline{1}6$ ) So  $24 \times 10 = 240$ . This is recorded under the line. The first number is then multiplied by the next most significant digit (in this case the units digit which is  $\underline{6}$ ) So  $24 \times 6 = 144$ . This is recorded under the line too. Once all of the multiplication calculations have been carried out, the separate answers are combined to find the total (in this case 240 + 144 = 384)

Again this method progresses to enable the children to solve larger calculations too.

# DIVISION

# Division - Mental Methods

Early division begins with sharing in practical activities. They are taught to understand division as sharing and grouping.

#### <u>Sharing</u>

Children count how many objects are in each group, recognise equal groups and share items out in play and problem solving.



Six sweets are shared equally between 2 people. How many sweets does each child get?



#### Grouping

If there are 8 children how many groups of 2 are there?



Children need to recognise that  $15 \div 3 =$  can mean 15 shared between 3 or How many lots of 3 are there in 15?

4 apples are packed into a basket. How many baskets can you fill with 12 apples?



Grouping in 4s

# Division - Informal Written Methods

#### Division using a number line (Introduced in Year 2)

A number line can be used to solve division calculations too.

Grouping can be shown more efficiently using a number line with repeated subtractions. To find the answer, count the number of jumps.e.g. to work out how many groups of 6 there are in 30, you would begin at the left hand side of a number line and count in steps of 6 until you reach 30. This would show that there a 5 groups of 6 in 30 as you would make 5 jumps.

Repeated subtraction using a number line (Introduced in Year 2)



Remainders

If a number can not be divided equally then there is a remainder. This same method can be used even if the calculation has a remainder. The children will be able to see if there is a remainder if in their final jump does not land on their target number as in the example below. The jumps which are of a consistent size of 5 can only get as close to 17 as 15, which took 3 jumps and there was a remainder of 2 as the next jump would land on 20 and that was too large.

17÷5= 3r2



The 'Chunking' method for division (Introduced in Year 3)

The 'Chunking' method for division is taught first using a number line



<u>Very</u> <u>Important</u> <u>Box!</u> 20x6=120 10x6=60 5x6=30 2x6=12 1x6=6 In order to save time making numerous jumps of 6 along the number line, larger 'chunks' can be jumped along the number line. A jump of 10 lots of 6 takes you to 60. Then you need another 4 lots of 6 to reach 84. Altogether that is **14** sixes.

When using a number line for the chunking method if there is a remainder in the calculation then the final 'chunk' would not land on the target number and the remaining amount would be the reminder, e.g.



#### Short Division (Introduced in Year 4)

An alternative way to record division calculations is vertically. The method below shows a vertical method.

To use this method of short division you divide the first number in the calculation by the second number, one digit at a time starting from the left.

So in this example firstly we divide the 9 (tens) by 7. This gives the result 1 (ten) remainder 2. The 1 is written in the tens position in the answer above the line. The remainder, 2 (tens) is then exchanged for 20 units. This exchange is indicated by the little 2 written in front of the 8. There are now 28 units to be divided by 7. This gives the result 4 (exactly), which is written above the line in the units position. The 14 written above the line (1 ten and 4 units) is therefore the answer to  $98 \div 7$ .



Where a number can not be divided equally there will be a remainder.

In the case of the example above there are only 4 (hundreds), not enough to divide by 5. Therefore we mentally exchange the 4 hundreds for 40 tens and now think of the number as 43 (tens) and 2 (units). So we are now looking at 43 (tens) to be divided by 5. The answer is 8 (tens) with 3 (tens) remaining. The **8** is written in the tens position above the line. The 3 (tens) remaining are exchanged for 30 units and a little 3 is written in front of the 2 units to indicate this. There are now 32 units to be divided by 5, which gives a result of 6 (units) with 2 remaining. The **6** (units) is written above the answer line in the units position in the answer. So  $432 \div 5 = 86r2$ 

#### Long Division (Introduced in Year 6)

	Н	Т	U	
		2	8	r 12
15	4	3	2	
	3	0	0	
	1	3	2	
	1	2	0	
		1	2	

There are only 4 (hundreds), not enough to divide by 15 so the 4 (hundreds) are mentally exchanged for 40 (tens) to now make 43 (tens). We can divide 43 (tens) by 15. This gives us 20 (tens) and 13 (tens) left over. We write the 2 above the line to represent the 20 (tens) and we note down what 20 lots of 15 is (300). We subtract 300 from 432 and are left with 132. We now only have 1 (hundred) which can't be divided by 15. We mentally exchange this for 13 (tens) but this still can't be divided by 15 so we then have to mentally exchange this for 132 (units). 132 (units) can be divided by 15 giving a result of 8 (units) and 12 (units) left over. The 8 is recorded above the line to form part of the answer. 8 lots of 15 is 120 so we record 120 under the 132 and subtract it leaving 12 as a remainder due to the fact that it is less than the number that we are dividing by.

432 ÷ 15 = 28r12

Long division can also be used to solve division calculations that include decimal numbers such as the example below.

432 ÷ 15 = 28.8

HTU.th

# Resources to aid solving calculations

## 100 Square

1	2	3	4	5	6	7	8	9	10
11	1 2	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

## Multiplication Square

Х	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

#### 0-100 Number line

